

# Mirror Symmetry via Deformation of Bundles on K3 Surfaces

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## ABSTRACT

We consider F-theory compactifications on a mirror pair of elliptic Calabi–Yau threefolds. This yields two different six-dimensional theories, each of them being nonperturbatively equivalent to some compactification of heterotic strings on a  $K3$  surface  $S$  with certain bundle data  $E \rightarrow S$ . We find evidence for a transformation of  $S$  together with the bundle that takes one heterotic model to the other.

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## 1. Introduction

An area that has been focus of extensive research in recent times is that of Heterotic/Type II duality, which was first studied in [1-4], where compactifications with 16 supersymmetry generators were considered. These results were extended to  $N = 2$ ,  $d = 4$  compactifications in [5,6], where the heterotic theory on  $K3 \times T^2$  was conjectured to be dual to the Type IIA theory on a Calabi–Yau threefold. This duality may be lifted to six dimensions upon going to the large radius limit of the torus (with the Wilson lines, if any, switched off), to obtain Heterotic/F-Theory duality in six dimensions [7,8], where the Calabi–Yau threefold is now an elliptic fibration. This raises an interesting question. Consider F-theory compactifications on a mirror pair of elliptic Calabi–Yau threefolds. This yields two different six-dimensional theories, each of them being nonperturbatively equivalent to some compactification of heterotic strings on a  $K3$  surface  $S$  with certain bundle data  $E \rightarrow S$ . Is there a simple transformation of  $S$  together with the bundle that would take one heterotic model to the other?

This is the issue that we address in this note. We argue that given a heterotic vacuum dual to F-theory compactified on an elliptic Calabi–Yau threefold  $\mathcal{M}$ , the heterotic dual of F-theory compactified on the mirror manifold  $\mathcal{W}$  is obtained by essentially exchanging the roles of large (*i.e.*, finite-sized) and small (*i.e.*, point-like) instantons. The non-perturbative phenomena associated with the appearance of small instantons were first studied in [9] and their results have been extended in [10-15]. In this paper, we will mainly be using the results of [16] which describes the enhanced gauge symmetry that results when small instantons coalesce onto orbifold singularities of  $K3$ .

The organization of this note is as follows. In §2, we state our proposal relating pairs of heterotic theories to mirror pairs of Calabi–Yau threefolds, and prove that the Hodge numbers are consistent with our hypothesis. In §3, we study some examples of mirror pairs and verify that the gauge and tensor multiplet content of the F-theory compactifications (which is obtained from the singularity structure of the Calabi–Yau threefolds [17-19]) matches that obtained on the heterotic side [16]. §4 summarises our results.

## 2. Deformations of bundles over $K3$ surfaces

In this section, we describe our proposal which relates F-theory compactifications on mirror pairs of Calabi–Yau threefolds and their heterotic duals. Our results are valid for elliptic Calabi–Yau threefolds whose mirrors are also elliptic fibrations.

### 2.1. The Proposal

Heterotic string theory compactified on an elliptic  $K3$ , with some appropriate choice of gauge bundle, yields a six dimensional theory with  $N = 1$  supersymmetry. This is conjectured to be dual to F-theory compactified on an elliptic Calabi–Yau threefold  $\mathcal{M}$ . Now consider F-theory compactified on the mirror manifold  $\mathcal{W}$ . Our proposal is that the heterotic dual of this model can be obtained from the original heterotic model by applying the following map:

- Large instantons in a gauge bundle with structure group  $H$  map to an equal number of small instantons sitting on a  $\mathbf{H}$  orbifold singularity of the  $K3$  and vice versa, where  $\mathbf{H}$  denotes both the subgroup of  $SL(2, \mathbb{Z})$  as well as the surface singularity corresponding to  $H$ .
- Small instantons sitting on a *smooth* point of the  $K3$  map to themselves.

For example, the heterotic  $E_8 \times E_8$  theory compactified on a  $K3$  with 24 (large) instantons in an  $E_8$  gauge bundle yields a six-dimensional theory with generic gauge group  $E_8$ . Its F-theory dual is obtained using the Calabi–Yau threefold  $\mathcal{M}$  with Hodge numbers  $h_{11} = 11$ ,  $h_{21} = 491$ . Compactifying F-theory on the mirror  $\mathcal{W}$  of this manifold yields a theory with 193 tensor multiplets and gauge group  $E_8^{17} \times F_4^{16} \times G_2^{32} \times SU(2)^{32}$  [17,18]. Using the map described above, we find that the heterotic dual is compactified on a  $K3$  with 24 small instantons on an  $E_8$  singularity, which was shown in Ref. [16].

### 2.2. Evidence for the proposal: matching Hodge numbers

The necessary condition for our proposal to work is that the heterotic theory obtained by applying the above map be dual to F-theory compactified on a manifold  $\mathcal{W}'$  with Hodge numbers which are precisely those of  $\mathcal{W}$ , *i.e.*,  $h_{11}(\mathcal{W}') = h_{21}(\mathcal{M})$  and  $h_{21}(\mathcal{W}') = h_{11}(\mathcal{M})$ .

This can be proved as follows. For definiteness, let us suppose that we start with a heterotic  $E_8 \times E_8$  model in which all the instantons are large and sitting in a bundle with structure group  $H \times H'$  (where  $H$  and  $H'$  are subgroups of the first and second  $E_8$  respectively), so that the gauge symmetry is  $G \times G'$ , where  $G$  and  $G'$  are the commutants in  $E_8$  of  $H$  and  $H'$ , respectively. Let us assume that there are  $k_1$  instantons in  $H$  and  $k_2$  in  $H'$ , with  $k_1 + k_2 = 24$ . The F-theory dual of this model is provided by a Calabi–Yau threefold  $\mathcal{M}$ . Then,

$$h_{11}(\mathcal{M}) = \text{rank}(G) + \text{rank}(G') + 3.$$

Applying the map, we find that the proposed mirror model has  $k_1$  small instantons on an  $\mathbf{H}$  orbifold singularity and  $k_2$  small instantons on an  $\mathbf{H}'$  orbifold singularity.  $h_{21}(\mathcal{W}')$  is the number of deformations of complex structure. Since all the instantons are now small, and sitting at special points, they do not contribute any moduli. Fixing the orbifold singularities in the  $K3$  results in a reduction of the number of hypermultiplet moduli (deformations) by an amount given by  $\text{moduli}(\mathbf{H}) + \text{moduli}(\mathbf{H}')$  (where  $\text{moduli}(\mathbf{H})$  is the number of moduli needed to specify the type  $\mathbf{H}$  singularity), so that we find

$$h_{21}(\mathcal{W}') = 20 - \text{moduli}(\mathbf{H}) - \text{moduli}(\mathbf{H}') - 1.$$

Now, one can verify (*e.g.*, by studying the branching rules), that  $\text{moduli}(\mathbf{H}) + \text{rank}(G) = \text{rank}(E_8) = 8$ . Thus we see that

$$h_{21}(\mathcal{W}') = h_{11}(\mathcal{M}).$$

Next, the number of deformations of complex structure of  $\mathcal{M}$  is equal to the number of hypermultiplet moduli of our original heterotic model minus one. The latter consists of the moduli of the gauge bundle and those of the  $K3$  itself. Recall that the dimension of the moduli space of  $k$  instantons in a group  $H$  equals  $h(H)k - \dim(H)$ , where  $h(H)$  denotes the dual Coxeter number of  $H$ . Also the  $K3$  is generic and hence provides us with 20 moduli. Thus,

$$h_{21}(\mathcal{M}) = h(H)k_1 - \dim(H) + h(H')k_2 - \dim(H') + 19.$$

On the other hand,

$$h_{11}(\mathcal{W}') = \text{rank}(\tilde{G}) + \tilde{n}_T + 2,$$

where  $\tilde{G}$  is the total gauge group and  $\tilde{n}_T$  is the total number of tensor multiplets resulting from putting  $k_1$  point-like instantons on a  $H$  singularity and  $k_2$  point-like instantons on a  $H'$  singularity of the  $K3$ . The outcome of such a process can be easily found using

the results of [16]. We give the results in Table 2.1. From this, it is clear that  $k$  small instantons on a  $\mathbf{H}$  orbifold singularity give rise to a gauge group  $\mathcal{G}$  and  $n'_T$  tensor multiplets such that

$$\text{rank}(\mathcal{G}) + n'_T = h(H)k - \dim(H).$$

In addition, we also obtain the primordial  $E_8 \times E_8$  gauge group (since there are no large instantons left) and one tensor multiplet, so that

$$h_{11}(\mathcal{W}') = h_{21}(\mathcal{M}).$$

Now consider the situation where there are small instantons sitting on *smooth* points of the  $K3$ . Smoothing out the singularity due to a small instanton requires one blowup, so we find that the contribution to  $h_{11}$  is one. Furthermore, a small instanton also yields a single hypermultiplet modulus, corresponding to its position, hence the contribution to  $h_{21}$  is also one. Therefore, mapping a small instanton on a smooth point of the  $K3$  to itself is consistent with mirror symmetry. Clearly, the proof above holds even when the heterotic vacuum consists of a mixture of small and large instantons.

Although the proof above considered only the case of  $E_8$  instantons, the same argument applies for simple  $\text{Spin}(32)/\mathbb{Z}_2$  instantons. Once again, by counting the number of tensor multiplets and gauge groups obtained when small  $\text{Spin}(32)/\mathbb{Z}_2$  instantons coalesce on an  $\mathbf{H}$  orbifold singularity, it is easy to see that  $h_{11}(\mathcal{W}') = h_{21}(\mathcal{M})$  (see Table 2.2). The case when more than one small  $\text{Spin}(32)/\mathbb{Z}_2$  instanton sits on a smooth point of the  $K3$  is a little subtle. While there is only one hypermultiplet modulus coming from the position of the small instantons, we obtain additional neutral hypermultiplets upon going to the Coulomb branch of the theory. This is because the gauge group is non-simply-laced ( $Sp(k)$  for  $k$  coincident small instantons), and the charged matter content consists of representations of  $Sp(k)$  which contain zero weight vectors, which yield exactly  $k - 1$  additional moduli when we go to the Coulomb branch.

$H$	$H$	$k$	$n'_T$	$\mathcal{G}$	$n'_T + \text{rank}(\mathcal{G})$
$SU(3)$	$A_2$	$\geq 5$	$k$	$SU(2) \times SU(3)^{(k-5)} \times SU(2)$	$3k - 8$
$SU(m)$	$A_{m-1}$	$\geq 2m$	$k$	$SU(2) \times SU(3) \times \dots \times SU(m-1) \times$ $SU(m)^{(k-2m+1)} \times \dots \times SU(2)$	$km - m^2 + 1$
$G_2$	$D_4$	6	6	$SU(2) \times G_2 \times SU(2)$	$4k - 14$
$G_2$	$D_4$	7	$2k-6$	$SU(2) \times G_2^2 \times SU(2)$	$4k - 14$
$SO(8)$	$D_4$	$\geq 7$	$2k-6$	$SU(2) \times G_2 \times SO(8)^{(k-7)}$ $\times G_2 \times SU(2)$	$6k - 28$
$SO(2m+7)$	$D_{m+4}$	$2m+6$	$2k-6$	$SU(2) \times G_2 \times SO(9) \times SO(3) \times \dots \times$ $SO(2m-1) \times SO(2m+7) \times$ $SO(2m-1) \times \dots \times SU(2)$	$2m^2 + 9m + 9$
$SO(2m+8)$	$D_{m+4}$	$\geq 2m+7$	$2k-6$	$SU(2) \times G_2 \times SO(9) \times SO(3) \times \dots \times$ $SO(2m-1) \times SO(2m+7) \times Sp(m) \times$ $(SO(2m+8) \times Sp(m))^{(k-2m-7)} \times$ $\dots \times SO(9) \times G_2 \times SU(2)$	$k(2m+6) -$ $2m^2 - 15m - 28$
$E_6$	$E_6$	$\geq 10$	$4k-22$	$SU(2) \times G_2 \times F_4 \times SU(3) \times$ $(E_6 \times SU(3))^{(k-9)} \times F_4 \times G_2 \times SU(2)$	$12k - 78$
$E_7$	$E_7$	$\geq 10$	$6k-40$	$SU(2) \times G_2 \times F_4 \times G_2 \times SU(2) \times E_7 \times$ $(SU(2) \times SO(7) \times SU(2) \times E_7)^{(k-10)}$ $\times SU(2) \times G_2 \times F_4 \times G_2 \times SU(2)$	$18k - 133$
$E_8$	$E_8$	$\geq 10$	$12k-96$	$E_8^{(k-9)} \times F_4^{(k-8)} \times$ $G_2^{(2k-16)} \times SU(2)^{(2k-16)}$	$30k - 248$

**Table 2.1:**  $k$  small  $E_8$  instantons on an  $H$  orbifold singularity. Note that each entry in the last column is equal to the dimension of the moduli space of  $k$  instantons in a gauge bundle with structure group  $H$ . Based on Table 2 of Ref. [16].

H	$k_{\min}$	$n'_T$	$\mathcal{G}$	$n'_T + \text{rank}(\mathcal{G})$
$A_{m-1}$ ( $m$ even)	$2m$	$\frac{m}{2}$	$Sp(k) \times SU(2k-8) \times SU(2k-16) \times \dots$ $\times SU(2k-4m+8) \times Sp(k-2m)$	$km - m^2 + 1$
$A_{m-1}$ ( $m$ odd)	$2m-2$	$\frac{m-1}{2}$	$Sp(k) \times SU(2k-8) \times SU(2k-16) \times \dots$ $\times SU(2k-4m+4)$	$km - m^2 + 1$
$D_{m+4}$ ( $m$ even)	$2m+8$	$m+4$	$Sp(k) \times Sp(k-8) \times SO(4k-16) \times$ $Sp(2k-16) \times \dots \times Sp(2k-4m-8)$ $\times SO(4k-8m-16) \times Sp(k-2m-8)^2$	$k(2m+6) -$ $\frac{1}{2}(2m+8)(2m+7)$
$D_{m+4}$ ( $m$ odd)	$2m+6$	$m+3$	$Sp(k) \times Sp(k-8) \times SO(4k-16) \times \dots$ $\times Sp(2k-4m-4) \times SO(4k-8m-8)$ $\times Sp(2k-4m-12) \times SU(2k-4m-12)$	$k(2m+6) -$ $\frac{1}{2}(2m+8)(2m+7)$
$E_6$	8	4	$Sp(k) \times SO(4k-16) \times Sp(3k-24)$ $\times SU(4k-32) \times SU(2k-16)$	$12k - 78$
$E_7$	12	7	$Sp(k) \times SO(4k-16) \times Sp(3k-24) \times$ $SO(8k-64) \times Sp(2k-20) \times Sp(3k-28)$ $\times SO(4k-32) \times Sp(k-12)$	$18k - 133$
$E_8$	11	8	$Sp(k) \times SO(4k-16) \times Sp(3k-24) \times$ $SO(8k-64) \times Sp(5k-48) \times SO(12k-112)$ $\times Sp(3k-32) \times Sp(4k-40) \times SO(4k-32)$	$30k - 248$

**Table 2.2:**  $k$  small  $\text{Spin}(32)/\mathbb{Z}_2$  instantons on an  $H$  orbifold singularity. Note that each entry in the last column is equal to the dimension of the moduli space of  $k$  instantons in a gauge bundle with structure group  $H$ . Based on Table 4 of Ref. [16].

### 3. Examples

We have shown that the Hodge numbers of the Calabi–Yau threefold  $\mathcal{W}'$  constructed by using the large/small instanton map and the heterotic/F-theory duality transformation are precisely those of the actual mirror  $\mathcal{W}$ . While constituting a rather strong evidence in favor of our conjecture, this fact however does not guarantee that  $\mathcal{W}'$  coincides with  $\mathcal{W}$ . To strengthen our position, we will now make use of toric methods which allow us, given a Calabi–Yau threefold realized as a hypersurface in a toric variety, to explicitly construct its mirror.<sup>1</sup> Our strategy is to take  $\mathcal{M}$  which provides the F-theory dual of a (perturbative) heterotic vacuum, construct its mirror  $\mathcal{W}$  torically, and compare it with  $\mathcal{W}'$  obtained by applying the large/small instanton map. Our task is considerably simplified by the fact that the mirrors which we need were constructed in [17], where their massless vector and tensor multiplet spectra were identified, and the physics of small instantons on orbifold singularities was worked out in [16]. Thus what we really need to do is to compare the results of these two references. We list below a few examples.

(1) As pointed out in §2, the Calabi–Yau threefold with Hodge numbers  $h_{11} = 491$  and  $h_{21} = 11$  has precisely the singularity structure that is obtained when 24 small  $E_8$  instantons coalesce onto an  $E_8$  singularity of the  $K3$ . On the other hand, this manifold is also the mirror of the Calabi–Yau threefold that yields the F-theory dual of the heterotic  $E_8 \times E_8$  model with 24 (large) instantons in a single  $E_8$  factor. We can actually go further and study the actual intersection pattern of the divisors corresponding to the blowups of the singularities. This was worked out for the heterotic model in Ref. [16]. For the Calabi–Yau threefold, this pattern can be read off from the dual polyhedron [17,19]. Once again, we find an exact match for this example and for all the other examples listed below.

(2) Next, consider the Calabi–Yau threefold with Hodge numbers  $h_{11} = 3$ ,  $h_{21} = 243$ . This corresponds to the heterotic model with 12 instantons in each  $E_8$  factor. The mirror manifold has Hodge numbers  $h_{11} = 243$  and  $h_{21} = 3$ , and gives gauge group  $E_8^8 \times F_4^8 \times G_2^{16} \times SU(2)^{16}$ , and 97 tensor multiplets. Now consider the heterotic model obtained from the map of §2. We have two  $E_8$  singularities with 12 small instantons on each of them. The total gauge group then consists of the primordial  $E_8 \times E_8$  in addition to two

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<sup>1</sup> To a Calabi–Yau hypersurface in a toric variety there corresponds a (reflexive) Newton polyhedron  $\Delta$  together with its dual  $\nabla$ , which is the Newton polyhedron of the mirror.



factors of  $E_8^3 \times F_4^4 \times G_2^8 \times SU(2)^8$ , and  $48 + 48 + 1 = 97$  tensor multiplets, in agreement with the previous result.

(3) Now unhiggs an  $SU(2)$  subgroup of  $E_8$  in the previous example. This yields a Calabi–Yau threefold with Hodge numbers  $h_{11} = 4$  and  $h_{21} = 214$ . On the heterotic side, this corresponds to 12 instantons in an  $E_7$  bundle and 12 more in an  $E_8$  bundle. From the toric data, we find that the mirror of the Calabi–Yau threefold gives 81 tensors and total gauge group  $E_8^5 \times E_7^3 \times F_4^6 \times G_2^{12} \times SO(7)^2 \times SU(2)^{16}$ . The heterotic model obtained from the map of §2 now gives a total gauge group consisting of the primordial  $E_8 \times E_8$  in addition to one factor of  $E_8^3 \times F_4^4 \times G_2^8 \times SU(2)^8$  and another factor of  $E_7^3 \times F_4^2 \times G_2^4 \times SO(7)^2 \times SU(2)^8$ , as well as  $48 + 32 + 1 = 81$  tensors.

(4) If we were to unhiggs  $SU(3)$  instead of  $SU(2)$  in the previous example, we would have to put the first 12 instantons in an  $E_6$  bundle. The Hodge numbers of the corresponding Calabi–Yau threefold are  $h_{11} = 5$  and  $h_{21} = 197$ . The toric data reveal that the mirror of this manifold gives 75 tensors and total gauge group  $E_8^5 \times E_6^3 \times F_4^6 \times G_2^{10} \times SU(3)^4 \times SU(2)^{10}$ . The heterotic model obtained from the map of §2 now gives a total gauge group consisting of the primordial  $E_8 \times E_8$  in addition to one factor of  $E_8^3 \times F_4^4 \times G_2^8 \times SU(2)^8$  and another factor of  $E_6^3 \times F_4^2 \times G_2^2 \times SU(3)^4 \times SU(2)^2$ , as well as  $48 + 26 + 1 = 75$  tensors.

(5) Now unhiggs an  $E_7$  subgroup of  $E_8$  in the example with 12 instantons in each  $E_8$ . This yields a Calabi–Yau threefold with Hodge numbers  $h_{11} = 10$  and  $h_{21} = 152$ . On the heterotic side, this corresponds to 12 instantons in an  $SU(2)$  bundle and 12 more in an  $E_8$  bundle. From the toric data, we find that the mirror of the Calabi–Yau threefold gives 61 tensors and total gauge group  $E_8^5 \times F_4^4 \times G_2^8 \times SU(2)^{17}$ . The heterotic model obtained from the map of §2 now gives a total gauge group consisting of the primordial  $E_8 \times E_8$  in addition to one factor of  $E_8^3 \times F_4^4 \times G_2^8 \times SU(2)^8$  and another factor of  $SU(2)^9$ , as well as  $48 + 12 + 1 = 61$  tensors.

(6) If we were to unhiggs  $E_8$  instead of  $E_7$  in the previous example, we would obtain a model with 12 extra tensor multiplets (coming from an equal number of small instantons sitting on smooth points of the  $K3$ ) and  $E_8$  gauge symmetry. The Hodge numbers of the corresponding Calabi–Yau threefold are  $h_{11} = 23$  and  $h_{21} = 143$ . The toric data reveal that the mirror of this manifold gives 61 tensors and total gauge group  $E_8^5 \times F_4^4 \times G_2^8 \times SU(2)^8$ . The heterotic model obtained from the map of §2 now gives a total gauge group consisting of the primordial  $E_8 \times E_8$  in addition to one factor of  $E_8^3 \times F_4^4 \times G_2^8 \times SU(2)^8$ , as well as  $48 + 12 + 1 = 61$  tensors, since the 12 small instantons map to themselves, giving 12 tensors and no additional enhancement of gauge symmetry.

(7) Consider the heterotic model with 8 instantons in an  $SO(8)$  gauge bundle and 16 instantons in an  $E_8$  gauge bundle. This gives a model with  $SO(8)$  gauge symmetry. The corresponding Calabi–Yau threefold has Hodge numbers  $h_{11} = 7$  and  $h_{21} = 271$ . The toric data reveal that the mirror manifold gives 107 tensors and gauge group  $E_8^9 \times F_4^9 \times G_2^{18} \times SU(2)^{18}$ . This appears to disagree with the heterotic result, which gives  $E_8^9 \times F_4^8 \times SO(8) \times G_2^{18} \times SU(2)^{18}$  instead. This is because we need to be more careful in identifying the groups from the toric data. Out of the nine  $F_4$  factors seen in the polyhedron, one is different from all the rest. The divisor corresponding to this factor has self-intersection  $-4$  rather than  $-5$ , meaning that there is charged matter in the **26** of  $F_4$  [19]. Exact agreement with the heterotic result can now be obtained by Higgsing the  $F_4$  to  $SO(8)$ . It is easy to see that the Hodge numbers remain unchanged during this process.

(8) Consider the heterotic model with 6 instantons in an  $SU(3)$  gauge bundle and 18 instantons in an  $E_8$  gauge bundle. This gives a model with  $E_6$  gauge symmetry. The corresponding Calabi–Yau threefold has Hodge numbers  $h_{11} = 9$  and  $h_{21} = 321$ . The toric data reveal that the mirror manifold gives 127 tensors and gauge group  $E_8^{11} \times F_4^{10} \times G_2^{21} \times SU(2)^{22}$ . Again, this disagrees with the heterotic result, which is  $E_8^{11} \times F_4^{10} \times G_2^{20} \times SU(3) \times SU(2)^{22}$ . Once again, we need to be careful in the analysis of the toric data. One of the  $G_2$ ’s is seen to be different from all the rest in that there is extra charged matter in the **7** of  $G_2$  [19], which can be used to Higgs the  $G_2$  down to  $SU(3)$ , to obtain exact agreement with the heterotic result.

(9) Finally, consider the self-mirror manifold with Hodge numbers  $(43, 43)$ . The corresponding heterotic model may be obtained in two ways. One is to consider the  $E_8 \times E_8$  model with 24 tensor multiplets, and the other is to consider the  $SO(32)$  model with 24 coincident small instantons which gives an additional  $Sp(24)$  gauge symmetry. It was argued in Ref. [16] that these models are T-duals of each other. In each case, the heterotic model maps to itself under the map of §2, in agreement with the fact that the manifold is self-mirror.

## 4. Discussion

In this note, we have proposed a map relating mirror symmetry in Calabi–Yau threefolds to heterotic compactifications. We have shown that the heterotic equivalent of the mirror map relates large instantons in a gauge bundle and small instantons on the corresponding orbifold singularity. We find that this map is consistent with the exchange of Hodge numbers under the mirror map, and have also shown that it is consistent with the singularity structure of the manifolds in a number of examples. This is strong evidence in favor of the proposal.

Although we have mainly considered the  $E_8 \times E_8$  heterotic theory, our results are applicable to the  $SO(32)$  theory also. While we have proved that the Hodge numbers match irrespective of whether the instantons are of  $E_8$  or  $\text{Spin}(32)$  type, comparing the singularity structure is a different matter. The singularity structure of the mirrors of the Calabi–Yau threefolds corresponding to  $SO(32)$  models are currently being worked out [20]. Preliminary results appear to corroborate our statements, though more examples need to be worked out. Of course, if we were to compactify further on a circle, the resulting T-duality between the  $E_8 \times E_8$  and  $SO(32)$  theories would suggest that our results should also hold for the  $SO(32)$  models.

These results should hopefully shed some light on the origins of mirror symmetry in Calabi–Yau manifolds. While our results hold only for elliptic Calabi–Yau threefolds whose mirrors are also elliptic fibrations (because F-Theory/Heterotic duality holds only for elliptic fibrations), perhaps a generalization of this approach works for all Calabi–Yau manifolds<sup>2</sup>.

## Acknowledgements

We wish to thank P. Candelas for reading a preliminary version of this paper and providing useful comments. This work was supported by the Robert Welch Foundation and NSF grant PHY-9511632.

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<sup>2</sup> It would be interesting to see how our results are related to the interesting proposal of [21] that mirror symmetry is in fact a form of T-duality.

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